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Heat and Mass Transfer Analysis due to Mixed Convection in a Rotating Fluid

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Abstract
This article studies the free convection flow of viscous fluid in a rotating frame. Both heat and mass transfer phenomenon are considered. Magnetohydrodynamic (MHD) fluid over an infinite disk is set into impulsive motion in the presence of thermal radiation and thermal diffusion effects. Exact solutions are obtained using the Laplace transform technique; satisfy all imposed initial and boundary conditions. These solutions are plotted graphically for velocity, temperature and concentration profiles in order to see the effects of the various parameters entering into the problem. The results show that with increasing rotation parameter primary velocity decreases whereas the absolute value of the secondary velocity increases.

Keywords
MHD, free convection, thermal diffusion , rotating fluid, exact solution.

1. Introduction

Study of rotating fluid flows has drawn considerable interest of researchers these days due to its wide range of applications in cosmological and geophysical fluid dynamics, astrophysics, meteorology, petroleum, designing thermo syphon tubes, in cooling turbine blades and in hydrology to study the movement of underground water. Having such motivation, Jana et al. [1] investigated the unsteady flow of viscous fluid through a porous medium bounded by a porous plate in a rotating system. The rotating flows are even more important in the presence of a magnetic field owing to their applications in sunspot development, solar cycle, rotating magnetic stars and in planetary atmosphere research. In view of these applications of the MHD rotating flows, very recently, Farhad et al. [2] extended the work of Jana et al. [1] to the electrically conducting viscous fluid with the consideration of Hall current and slip condition at the boundary. Two other recent attempts considering MHD rotating flows are made by Sahoo et al. [3] and Seth et al. [4].

Despite of the fact that free convection flows are greatly influence by rotation, yet very little attention is given to it in the literature. The Coriolis force, which is a result of a motion of rotating frame, is responsible for the production of rotation in the geofluid. Chandrasekhar [5] pointed out the significant role of the Coriolis force on problems of thermal instability and on stability of viscous flow has made significant contributions to the theory of hydrodynamic flow phenomena in numerous situations. Then, several others studies are carried out by various investigators disclosed that Coriolis force is significant compared to viscous and inertia occurring in the basic equations of the problems involving unsteady free-convection flow past an infinite vertical plate in rotating fluid. Amongst them, the important studies in this direction are made by Singh [6], Bestman and Adjepong [7], Singh et al. [8], Lahurikar [9], Vijayalakshmi[10,11], Nanousis [12], and Mbeledogu and Ogulu [13].

The present article aims to extend the work of Vijayalakshmi [10] to the case of electrically conducting fluid with the inclusion of mass transfer and the thermal diffusion effect as well. Exact solutions of the governing partial differential equations are obtained using the

Laplace transform technique. The influence of the embedded parameters on the flow, thermal and concentration fields are graphically presented and analyzed.

2. Mathematical Analysis

We consider unsteady MHD flow of viscous fluid with combined phenomenon of heat and mass transfer are considered together with thermal radiation and thermal diffusion effects. Both the plate and the fluid are in a state of rigid body rotation with uniform angular velocity Ω about *z*-axis. A uniform magnetic field of strength B_0 is acting perpendicular to the flow i.e along *z*-axis in outward direction. Initially ($t \le 0$), the disk and the fluid are at rest such that the free stream temperature T_∞ and concentration C_∞ are the same everywhere. At time $t > 0$, the disk starts moving in its own plane with a constant velocity U_0 along the *x*-axis. At the same time, the temperature of the plate and concentration are become T_w and C_w . Under these assumptions and using Boussinesq approximation, this problem is governed by the following equations (Nanousis[12] and Vijayalakshmi [11])

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) - \frac{\sigma B_0^2}{\rho} u' \tag{1}$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho} v' \tag{2}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} + 16 a^* \sigma_1 T_\infty^3 (T_\infty - T') \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} + \frac{D K_T}{T_m} \frac{\partial^2 T'}{\partial z'^2} \tag{4}$$

where υ is the kinematic viscosity, t is the time, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of mass expansion, σ is the electrical conductivity, B_0 is the applied magnetic field, ρ is the fluid density, C_p is the specific heat of the fluid at constant pressure, k is the thermal conductivity of the fluid, D is the mass diffusivity, K_T is the thermal-diffusion ratio, T_m is the mean fluid temperature, a^* is the absorption coefficient and σ_1 is the Stefan-Boltzmann constant·

Initial and boundary conditions corresponding to this problem are

$$\begin{array}{lllll} t' \le 0 & u' = 0, & T' = T_\infty, & C' = C_\infty & \text{for all } z' \\ t' > 0 & \begin{cases} u' = U_0, \\ u' = 0, \end{cases} & \begin{array}{l} T' = T_w, \\ T' \to T_\infty, \end{array} & \begin{array}{l} C' = C_w \\ \mathrm{C}' \to \mathrm{C}_\infty \end{array} & \begin{array}{l} \text{at } z' = 0 \\ \text{as } z' \to \infty. \end{array} \end{array} \tag{5}$$

Introducing these dimensionless variables

$$\begin{array}{lll} u, v = \dfrac{u', v'}{U_0}, & z = \dfrac{z' U_0}{\upsilon}, & t = \dfrac{t' U_0^2}{\upsilon}, \\ T = \dfrac{T' - T_\infty}{T_w - T_\infty}, & C = \dfrac{C' - C_\infty}{C_w - C_\infty}, & \end{array} \tag{6}$$

into equations (1)-(5), we get

$$\frac{\partial q}{\partial t} + 2Eqi = \frac{\partial^2 q}{\partial z^2} + G\theta + GrC - mq, \tag{7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 T}{\partial z^2} + \frac{R}{\mathrm{Pr}} T, \tag{8}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2} + So \frac{\partial^2 C}{\partial z^2}, \tag{9}$$

$$\begin{array}{lllll} t \le 0 & q = 0, & \theta = 0, & C = 0 & \text{for all } z \\ t > 0 & \begin{cases} q = 1, \\ q = 0, \end{cases} & \begin{array}{l} T = 1, \\ T \to 0, \end{array} & \begin{array}{l} C = 1 \\ C \to 0 \end{array} & \begin{array}{l} \text{at } z = 0 \\ \text{as } z \to \infty, \end{array} \end{array} \tag{10}$$

where

$$q = u + iv, E = \frac{\Omega \upsilon}{U_0^2}, G = \frac{\upsilon g \beta (T_w - T_\infty)}{U_0^3}, Gr = \frac{\upsilon g \beta^* (C_w - C_\infty)}{U_0^3},$$

$$m = \frac{\upsilon \sigma B_0^2}{\rho U_0^2}, \mathrm{Pr} = \frac{\rho \upsilon C_p}{k}, R = \frac{16 a^* \upsilon^2 \sigma T_\infty^3}{k U_0^2}, Sc = \frac{\upsilon}{D}, So = \frac{D K_T}{T_m} \frac{(T_w - T_\infty)}{\upsilon (C_w - C_\infty)},$$

rotation parameter, Grashof number, modified number, magnetic parameter, Prandtl number, radiation parameter, Schmidth number and Soret number respectively.

Solutions of equations (7)-(9) subject to the initial and boundary conditions (13) by the Laplace transform method are

$$\theta(z,t) = \varphi(0, \frac{R}{\mathrm{Pr}}, z\sqrt{\mathrm{Pr}}), \tag{11}$$

$$C(z,t) = (1+A_1)\varphi(0,0,z\sqrt{Sc}) + A_2\,\varphi(-\frac{R}{\mathrm{Pr}-Sc},0,z\sqrt{Sc}) - A_1\varphi(0,\frac{R}{\mathrm{Pr}},z\sqrt{\mathrm{Pr}}) - A_2\varphi(-\frac{R}{\mathrm{Pr}-Sc},\frac{R}{\mathrm{Pr}},z\sqrt{\mathrm{Pr}}), \tag{12}$$

$$\begin{aligned} q = {} & (1+A_3-A_4)\varphi(0,a,z) - (A_3+A_5)\varphi(-\frac{R-a}{\mathrm{Pr}-1},a,z) + (A_4+A_5) \\ & \varphi(\frac{a}{Sc-1},a,z) - A_3\varphi(0,\frac{R}{\mathrm{Pr}},z\sqrt{\mathrm{Pr}}) + (A_3+A_5)\varphi(-\frac{R-a}{\mathrm{Pr}-1},\frac{R}{\mathrm{Pr}},z\sqrt{\mathrm{Pr}}) - \\ & A_5\varphi(-\frac{R}{\mathrm{Pr}-Sc},\frac{R}{\mathrm{Pr}},z\sqrt{\mathrm{Pr}}) + A_4\varphi(0,0,z\sqrt{Sc}) - (A_4+A_5) \\ & \varphi(\frac{a}{Sc-1},0,z\sqrt{Sc}) + A_5\varphi(-\frac{R}{\mathrm{Pr}-Sc},0,z\sqrt{Sc}), \end{aligned} \tag{13}$$

where

$$\varphi(x,y,z) = \frac{e^{xt}}{2}[e^{-z\sqrt{x+y}}erfc\{\frac{z}{2\sqrt{t}} - \sqrt{(x+y)t}\} + e^{z\sqrt{x+y}}erfc\{\frac{z}{2\sqrt{t}} + \sqrt{(x+y)t}\}],$$

$$a = m + 2Ei, \quad A_1 = SoSc, \quad A_2 = (\frac{SoSc\,\mathrm{Pr}}{(\mathrm{Pr}-Sc)} - SoSc),$$

$$A_3 = \frac{G - GrSoSc}{R-w}, \quad A_4 = \frac{Gr(1+SoSC)}{w},$$

$$A_5 = \frac{SoSc^2Gr}{R(Sc-1) + w(\mathrm{Pr}-Sc)}$$

and erfc(x) represents the complementary error function.

3. Skin Friction, Nusselt Number and Sherwood Number

$$\begin{aligned}\tau = (&-\frac{e^{-at}}{\sqrt{\pi t}} - \sqrt{a}erf\{\sqrt{at}\}) + \frac{G - GrScSo}{R-a}(\sqrt{R}erf\{\sqrt{\frac{R}{\Pr}t}\} \\ &- e^{-\frac{R-a}{\Pr-1}t}\sqrt{\frac{a\Pr - R}{\Pr-1}}erf\{\sqrt{(\frac{R}{\Pr} - \frac{R-a}{\Pr-1})t}\} \\ &+ e^{-\frac{R-a}{\Pr-1}t}\sqrt{\frac{a\Pr - R}{\Pr-1}}erf\{\sqrt{\frac{a\Pr - R}{\Pr-1}t}\} - \sqrt{a}erf\{\sqrt{at}\}) \\ &+ (\frac{GrSc^2So}{a(\Pr - Sc) + R(-1+Sc)})(\mathrm{e}^{-\frac{R-a}{\Pr-1}t}\sqrt{\frac{a\Pr - R}{\Pr-1}}erf\{\sqrt{(\frac{a\Pr - R}{\Pr-1})t}\} \\ &- \mathrm{e}^{-\frac{R-a}{\Pr-1}t}\sqrt{\frac{a\Pr - R}{\Pr-1}}erf\{\sqrt{(\frac{R}{\Pr} - \frac{-a+R}{-1+\Pr})t}\} \\ &- \mathrm{e}^{\frac{at}{Sc-1}}\sqrt{\frac{aSc}{Sc-1}}erf\{\sqrt{(\frac{aSc}{Sc-1})t}\} - \mathrm{e}^{-\frac{R}{\Pr-Sc}t}\sqrt{-\frac{RSc}{\Pr-Sc}}erf\{\sqrt{-\frac{Rt}{\Pr-Sc}}\} \\ &+ \mathrm{e}^{-\frac{R}{\Pr-Sc}t}\sqrt{-\frac{ScR}{\Pr-Sc}}erf\{\sqrt{(\frac{R}{\Pr} - \frac{R}{\Pr-Sc})t}\} \\ &+ \mathrm{e}^{\frac{a}{Sc-1}t}\sqrt{\frac{aSc}{Sc-1}}erf\{\sqrt{\frac{a}{Sc-1}t}\}) \\ &+ \frac{Gr(1+ScSo)}{a}(-\mathrm{e}^{\frac{a}{Sc-1}t}\sqrt{\frac{aSc}{Sc-1}}erf\{\sqrt{(\frac{aSc}{Sc-1})t}\} \\ &+ \mathrm{e}^{\frac{a}{Sc-1}t}\sqrt{\frac{aSc}{Sc-1}}erf\{\sqrt{\frac{a}{Sc-1}t}\} + \sqrt{a}erf\{\sqrt{at}\}).\end{aligned} \tag{14}$$

$$Nu = -\frac{\sqrt{\Pr}e^{-\frac{R}{\Pr}t}}{\sqrt{\pi t}} - \sqrt{R}erf\{\sqrt{\frac{R}{\Pr}t}\}, \qquad (15)$$

$$\begin{aligned}Sh = \frac{\sqrt{Sc}}{\sqrt{\pi t}} &- ScSo(\sqrt{R}erf\{\sqrt{\frac{R}{\Pr}t}\} - e^{-\frac{R}{\Pr-Sc}t}\sqrt{\frac{ScR}{Sc-\Pr}}erf\{\sqrt{(\frac{R}{\Pr} - \frac{R}{\Pr-Sc})t}\} \\ &+ e^{-\frac{R}{\Pr-Sc}t}\sqrt{\frac{RSc}{Sc-\Pr}}erf\{\sqrt{(\frac{R}{Sc-\Pr})t}\}) \\ &- (\frac{\Pr ScSo}{\Pr-Sc})(e^{-\frac{R}{\Pr}t}\sqrt{\frac{\Pr}{\pi t}} + e^{-\frac{R}{\Pr-Sc}t}\sqrt{\frac{ScR}{Sc-\Pr}}erf\{\sqrt{(\frac{R}{\Pr} - \frac{R}{\Pr-Sc})t}\} \\ &- \sqrt{\frac{Sc}{\pi t}} - e^{-\frac{R}{\Pr-Sc}t}\sqrt{\frac{RSc}{Sc-\Pr}}erf\{\sqrt{(\frac{R}{Sc-\Pr})t}\}).\end{aligned} \tag{16}$$

4. Graphical Results and Discussion

Graphical results are plotted to see the behavior of various embedded parameters on non-dimensional velocity components u and v, temperature T and concentration C. The Prandtl number Pr=0.71, corresponds to air, is used throughout numerical computation. Moreover, the Grashof number G and the modified Grashof number Gr are chosen equal to 10 and 5 respectively. On the other hand, the value of the Schmidt number Sc is taken as 0.22, which corresponds to H_2, with air as the solvent and the magnetic parameter m=1 is selected. The values of the Radiation parameter R, Soret parameter So and Rotation parameter E are chosen arbitrarily.

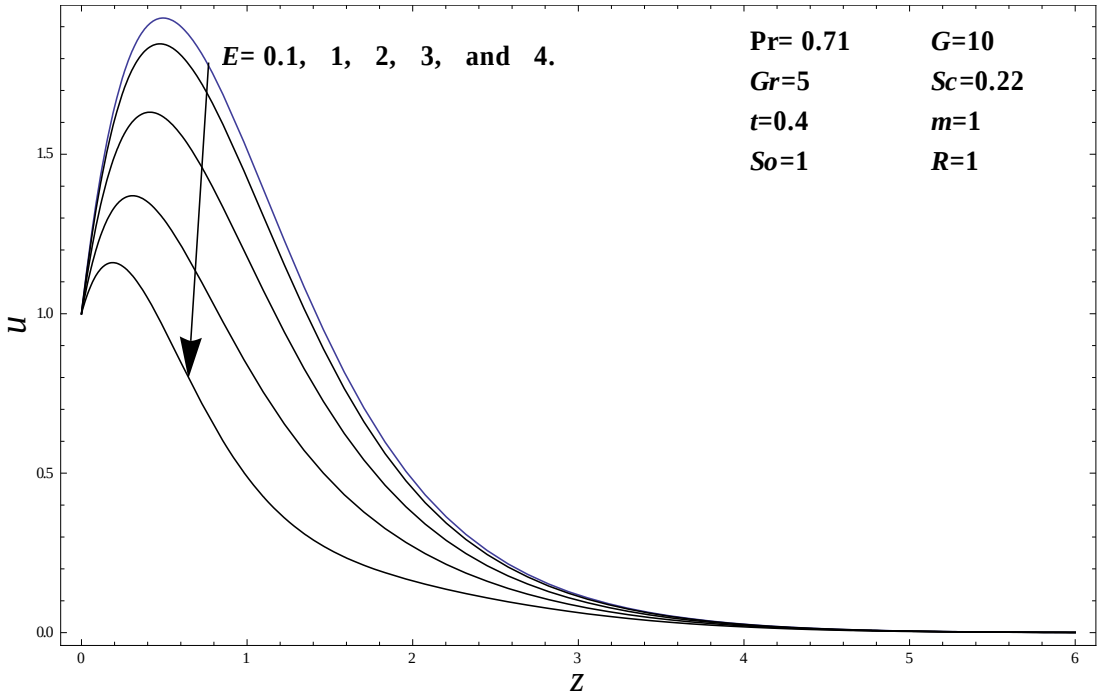

Figure 4.1(a) Primary velocity profile u against z for different rotation parameter, E.

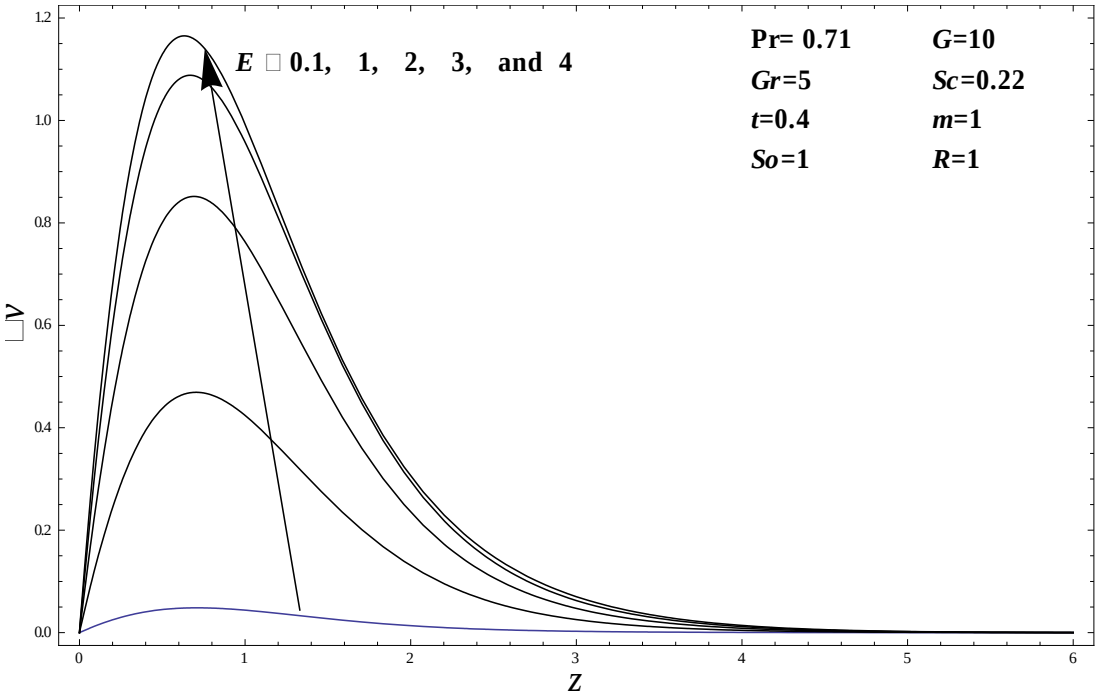

Figure 4.1(b) Secondary velocity profile *v*against *z* for different rotation parameter, *E*.

Figure 4.1(a) and Figure 4.1(b) illustrate that the primary velocity *v* decreases whereas the absolute value of the secondary velocity increases near the plate and tend to zero away from the plate with an increase in rotation parameter. This implies that rotation retards the primary velocity whereas it accelerates the secondary velocity. This may be attributed to the fact that when the frictional layer near the plate is suddenly set into motion then the Coriolis force acts as a constraint in the main fluid flow to induce cross flow in the flow field. Figure 4.1(b) shows that the influence of the rotation parameter *E* on the secondary velocity is significant near the plate. A small increase in the value of *E* will cause the secondary velocity to decrease rapidly.

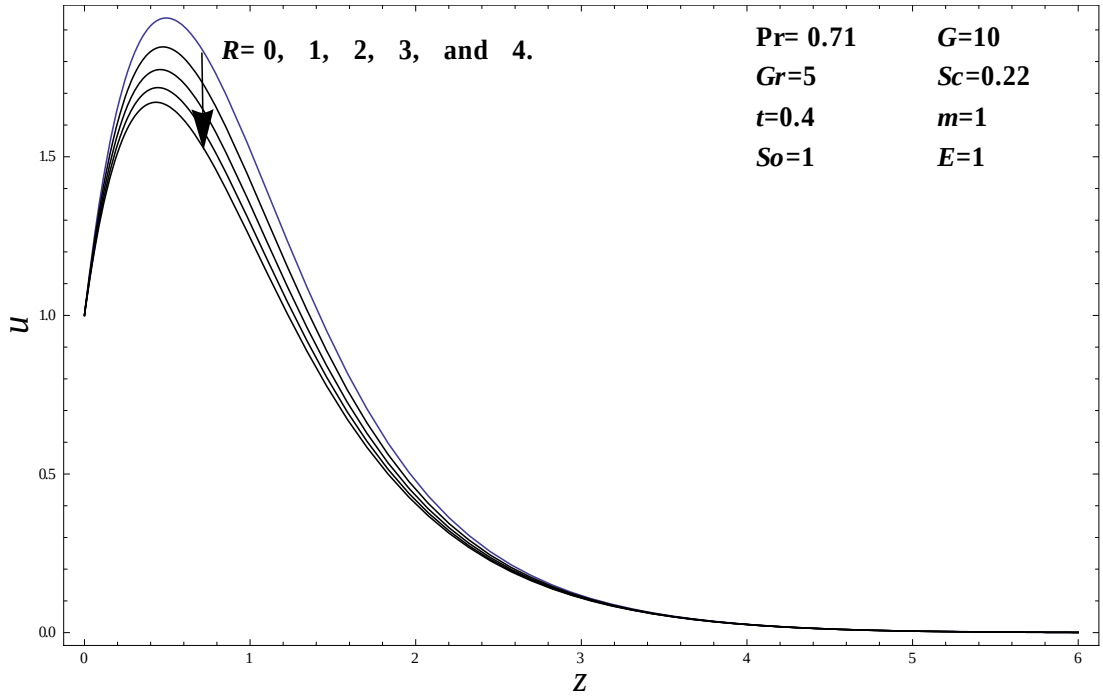

Figure 4.2(a) Primaryvelocity profile *u* against *z* for different radiation parameter, *R*.

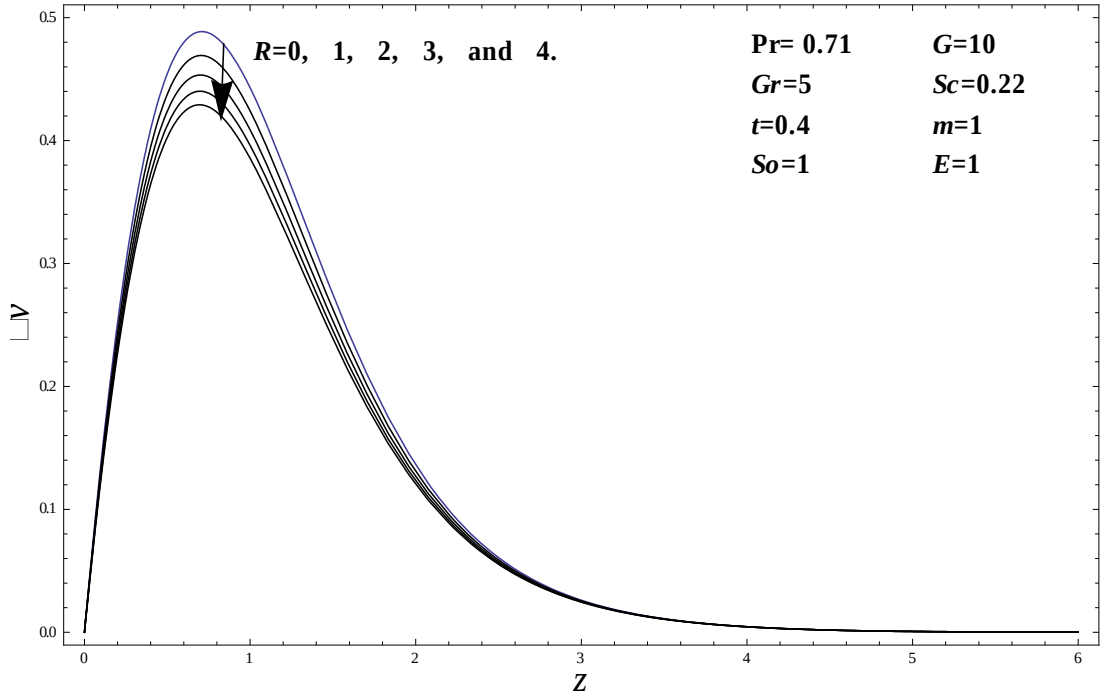

Figure 4.2(b) Secondary velocity profile *v* against *z* for different radiation parameter, *R*.

Figure 4.2(a) shows that with an increase inthe radiation parameter*R*,decreases the primary velocity*u*in the presence of radiation as well as in the case of pure convection.

Numerically for pure convection we take $R \to 0$, i.e. the radiation effectis negligible. Moreover, the absolute value of the secondary velocity in the presence of radiation as well as in the case of pure convection, decreases near the plate for increasing values of the radiation parameter R as can be seen from Figure 4.2(b).Moreover, t is observed that in case of pure convection $R \to 0$, the momentum boundary layer is greater results in a greater velocity and approaches to zero asymptotically. This fact is in accordance with the physical situation thatincreasing the radiation parameter contributes in the thickening of boundary layer.

The temperature profiles for air, Pr=0.71 are calculated for different values of radiation parameter R from equation (14) with time, t=0.4 and are presented in Figure 4.4. It is observed from this figure that temperature and thermal boundary layer thickness decrease with increasing radiation parameter$R$. Physically, it is true due to the fact that increasing radiation parameter R makes the fluid thicken, ultimately causing the temperature and the thermal boundary layer thickness to decrease.

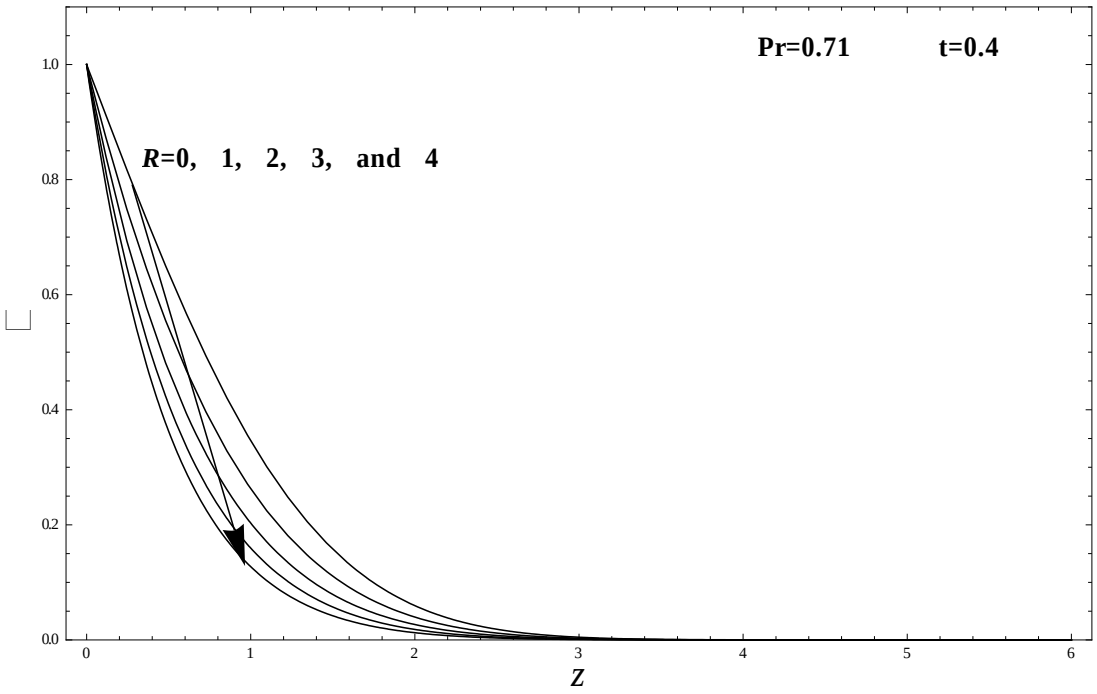

Figure 4.3 Profile of non-dimensional temperature θ for different values of radiation parameter R.

Effect of different Soret number on the concentration profiles is shown in Figure 4.5. We observed that the influence of the Soret number on the concentration profiles is significant. Concentration profiles increase with increasing values of the Soret number.

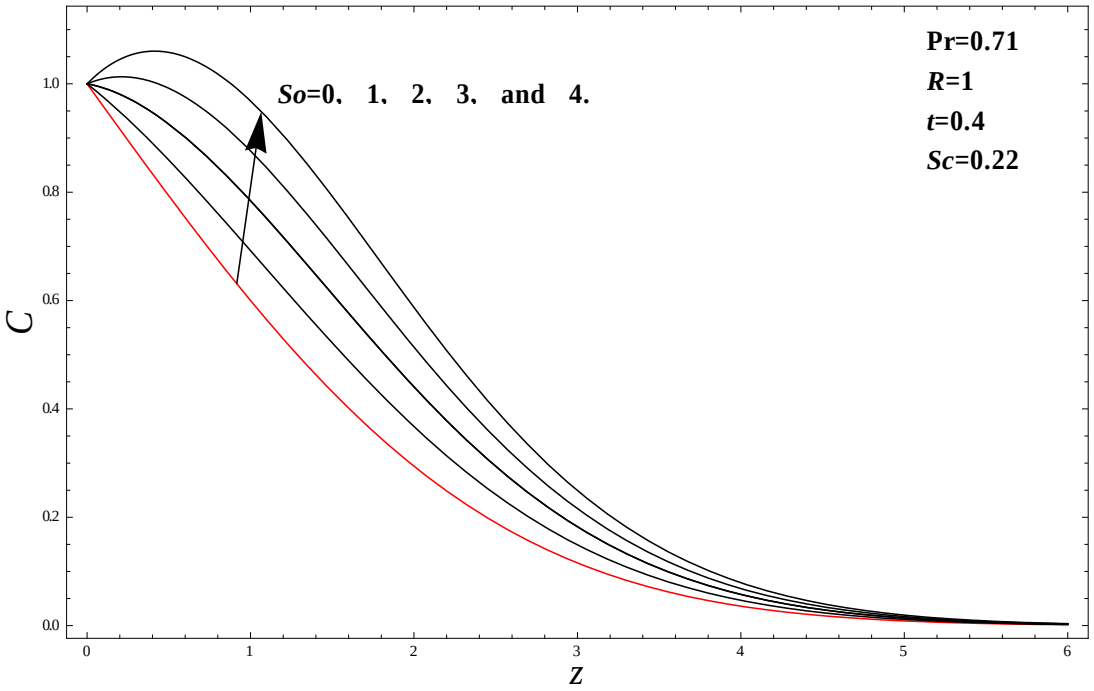

Figure 4.4 Profile of non-dimensional concentration *C* for different values of Soret number *So*.

5. Conclusion

Exact solutions for the rotating MHD free convection flow of viscous fluid with heat and mass transfer over an impulsively started infinite vertical plate in the presence of thermal radiation and thermal diffusion are obtained.It is found that they satisfy all imposed initial and boundary conditions and can be reduce as a special case to other well established results in the literature. From the graphical results of the fluid velocity, temperature and concentration, we found that the embedded parameters have strong influence on the present problem.

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